The Federal Student Loan Program: Quantitative Implications for College Enrollment and Default Rates

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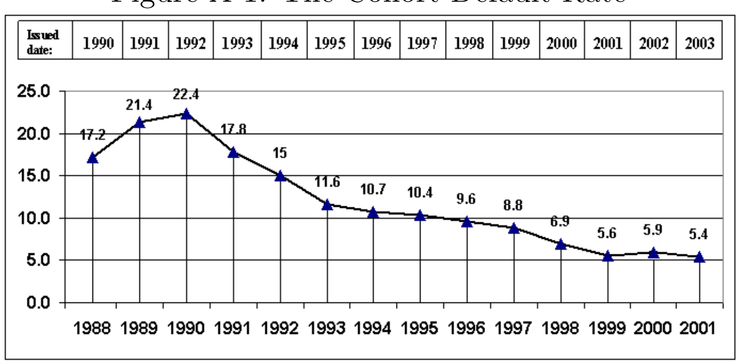
A Appendix

A.1 Data Details

Figure A-1 presents the cohort default rate for student loans (CDR) from 1988 to 2001. The CDR represents the percentage of borrowers who entered repayment in a fiscal year and defaulted by the end of the next year.

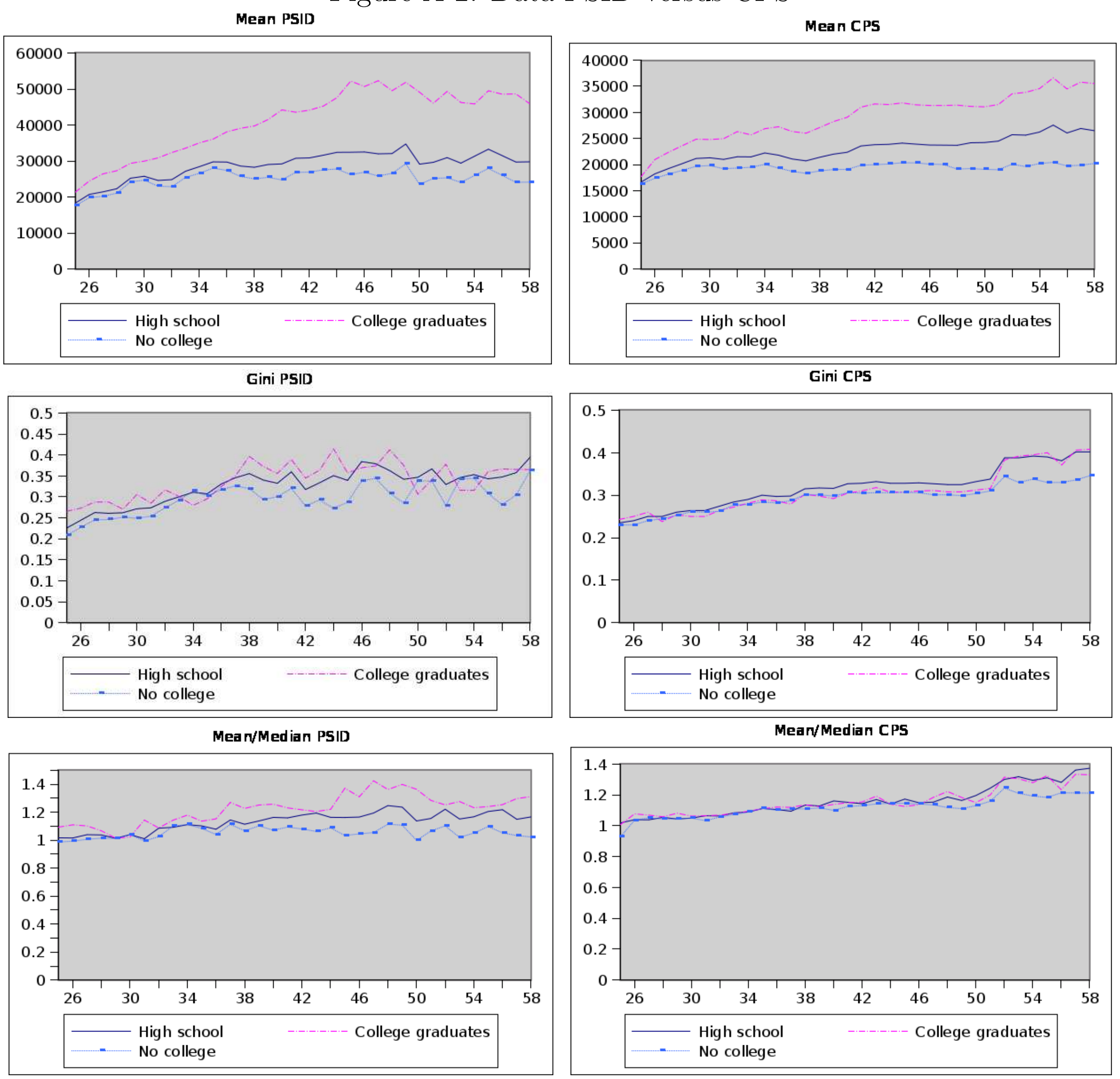
Figure A-1: The Cohort Default Rate



Source: Department of Education

I compute statistics for lifetime earnings based on earnings data from the CPS for 1969-2002 with synthetic cohorts. For each year in the CPS, I use earnings of heads of households age 25 in 1969, age 26 in 1970, and so on until age 58 in 2002. I consider a five-year bin to allow for more observations, i.e., by age 25 at 1969, I mean high school graduates in the sample that are 23 to 27 years old. Real values are calculated using the CPI 1982-1984. There are an average of 5000 observations in each year's sample. I divide education based on years of education completed with exactly 12 years for high school and those with 16 years of completed schooling for college. I also construct similar profiles using PSID family files for people age 25 in 1969 and followed until 2002 using the five year bin. The PSID sample includes 229 high school graduates. Among these, 49 have a college degree. The sample in the PSID is constructed similarly to the samples from the CPS using the five-year bin for age.[1] Figure A-2 presents the statistics constructed using the PSID data and those from the CPS. The statistics across the two samples are similar. I use the CPS data to construct earnings profiles to estimate the distribution of initial characteristics in my model, ability and initial human capital stock, since the sample size of the PSID is relatively small since it consists of only one cohort.

Figure A-2: Data PSID versus CPS



[1] There are no data on labor earnings for the years 1993-1995/2000, so I construct these using variables for the wages/salaries of household heads from main job, extra job, bonuses, tips, overtime, income from professional practice or trade, labor part of income from farm, business, market gardening, and room and boarders. There are no interviews for years 1998, 2000 and 2002, so data are missing for labor earnings for the years 1997, 1999 and 2001. I use linear interpolation on those years when constructing life-cycle earnings profiles.

A.2 Computation Procedure

1. I solve for the optimal decision rules for each education choice. For both paths, these include optimal time allocation for human capital/work and savings. On the college path, I additionally solve for optimal borrowing and repaying decisions. To calculate the optimal decision rules, I set a grid on learning ability, initial human capital, and initial assets (a, h, x) and compute life-cycle profiles of human capital, hours, and earnings from these grid points.
2. Given the optimal decision rules, I compute the stream of earnings for the two education groups from the model using appropriate parameters values. I also solve for optimal college enrollment for every (a, h, x) combination in the state space.
3. I choose the initial distribution of the state variable to best replicate the properties of U.S. data documented in Figure A-2. I find the parameter vector, $\gamma = (\mu_a, \sigma_a, \mu_h, \sigma_h, \rho)$, that characterizes the joint initial distribution for high school agents so as to minimize the distance between the model and the data (CPS) statistics for earnings mean, dispersion, and skewness. To recover the joint (a,h) distribution for high school graduates, I use the earnings for the two education groups and the optimal enrollment decision computed in step 2 and the exogenous initial assets distribution.

Computation algorithm:

1. To calculate the optimal decision rules in step 1, for any value of learning ability in the grid $a \in [0, a^*]$, which consists of 20 points, I put a non-uniform grid of 80 points on human capital, $[0, h^*]$, a grid of 15 points on assets, $[0, \overline{x}]$, and a grid of 30 points on debt, $[0, \overline{d}]$; the choices of h^* and a^* may be revised depending on the results of step 3.
a. For the no-college path, I compute the optimal decision rule for human capital and savings at grid points starting from period $j = J-1$, by solving the dynamic programming problem starting from period $J-1$, given $V_J(a, h) = w_J h$. Since the value function is concave in human capital each period, the dynamic programming problem is a concave programming problem. To calculate $h_j(a, h, x)$ at grid points, I compute the value function off grid points using linear interpolation.
b. For the college path, given the complexity of the computation, I break the problem into smaller sub-problems. First, I consider the post-college period, which is divided into two parts: post-payment and payment periods. For the post-payment period, the procedure is similar to that for the no-college path, given the same type of choices. The payment period supposes a more elaborate approach given the enlarged state space, multiple payment options, and stochastic rates. I solve for the optimal decision rule for human capital, $h_j(a, h, x, d, r)$ for each payment case. Backward recursion on Bellman's equations produces h_j for j = 5; ....,I with a different terminal time I for each payment option. Given decision rules for each payment path, I aggregate the results and choose the optimal payment as long as the agent does not choose to change from the standard fluctuating payments status. Once he switches, I consider the rules associated with

that particular path; otherwise I continue under the no consolidation status until full payment. Additionally, I keep track of the switching period and the payment choice to ease the computations later on. Given that the last period of payment depends on the chosen payment plan, I dynamically pick the relevant terminal node from the post-payment problem on a case-by-case basis. An additional problem that arises when solving for the payment period after college is the non-concavity of the value function on the no consolidation path, V^{NoC} (Equation 15 in the paper). This is the maximum over four continuation value functions. Even if these four value functions are strictly concave, the maximum over them is not. This can make the default behavior very sensitive to changes in the state variables, such as debt and human capital, which in turn makes finding the equilibrium default premium problematic, as in Chatterjee et al. (2007). Since I am not solving for equilibrium prices, I do not have the latter problem. However, it may still be the case that my results (in terms of the frequency of default) may be sensitive to the fineness of the grid. A solution for overcoming the non-concavity problem when there is discrete choice is adding idiosyncratic earnings shocks as implemented in Gomes et al. (2001) and Chatterjee et al. (2007). In my computation, however, there are important differences: 1) I search for the solution over the grid using backward induction on the value function, and 2) Once the switch to another repayment option occurs, the agent cannot switch back. There is no convergence or price iteration involved in my algorithm. I simply use a grid-search solution, and thus in my setup it is sufficient to check that the maximum of the continuation value function is unique during the period of the switch and that the solution is not sensitive to the fineness of the grid. I run a finer grid (100 points on human capital compared to 80 points) and I also check for the uniqueness of the solution. I find that the solution is not sensitive to the choice of the grid and that the maximum is unique.

The last step of part b involves solving for optimal decision rules during college periods. This implies optimal time allocation and savings. Additionally, in the first period, I solve for the optimal borrowing decision. I take the first period value function from the post-college problem as a terminal node for the college periods and solve using backward induction. After endogenizing the borrowing decision, I get the optimal decision rules for the initial state vector (a, h, x).

Solving this problem requires a tremendous amount of time. First, given the state space, I face the so-called "curse of dimensionality" (the grid consists of a total of 1,440,000 points). To avoid this problem, I solve for each level of ability separately. This is possible given that learning ability remains fixed over the agent's life. Second, the evolution of both debt and human capital are intrinsically determined by stochastic rates and optimal payment and time allocation decisions the agent makes. This imposes double interpolation with respect to debt and human capital, which can take a huge amount of time. To optimize on this, I implement the multi-linear interpolation method as described by John Rust (Amman et al. (1996)). I use simple piecewise linear interpolation of the value function in each coordinate of the state vector. To carry out the procedure, I impose an underlying grid on the state space defined by a Cartesian product of unidimensional grids over each coordinate of the state vector. I estimate the value function at an arbitrary point in the state space as a linear combination of the values of the function at the vertices of the grid hypercube containing the arbitrary point. This allows me to carry out the double interpolation procedure in a fraction of the time needed using the Matlab function. Third, after solving for optimal rules, I use the optimal borrowing choice to restrict my attention to the necessary data. I ignore the irrelevant debt levels and the subsequent choices to economize on stored data. I aggregate data for all ability levels and store the decision rules for the initial state space vector, given the determined payment choices and switching periods. Overall, these improvements reduced the computation time by 70%.

2. In step 2, I use the grid of 20 points on the ability grid, $[0, a^*]$, 15 points on the asset grid, $[0, \overline{x}]$, and extract 20 points out of the 80 points on the human capital grid, $[0, h^*]$.[2] This implies a total of 6000 points (a, h, x). Using the decision rules from step 1, I simulate life-cycle profiles of labor earnings from any initial pair (a, h, x) for each education choice and I solve for the optimal enrollment decision.

3. In step 3, I use the Matlab function "fminsearch", which implements the simplex algorithm to find the 400 values of the histogram over $[0, a^*] \times [0, h^*]$ that minimizes the distance between model and data statistics. For any trial of the vector describing the initial distribution, I calculate the mean, dispersion and skewness statistics at each age using the calculated life-cycle profiles and the guessed initial distribution. If the histogram that best matches the data puts strictly positive weight on (a,h) pairs where $a = a^*$ and $h = h^*$, then the upper bounds are increased and steps 1-3 are repeated.

A.3 Institutional Details on Ben-Porath: Implications for Initial Distribution

To study the significance of college enrollment and institutional details for the initial distribution, I compare the characteristics of the joint distribution of learning ability and initial human capital stock within my model to those derived within the standard Ben-Porath framework. I redo the calibration of the joint initial distribution within Ben-Porath in Huggett et al. (2006) restricted to high school graduates in my PSID sample.[3] Table A-1 presents the findings for both models. Note that lower levels of human capital and higher levels of ability, on average, are needed to match life-cycle earnings of high school graduates within my framework relative to Ben-Porath.

[2]Solving for decision rules requires a sufficiently fine grid for human capital, whereas finding the distribution of initial characteristics does not (for detailes see Huggett et al. (2006)).

[3]I use the PSID sample for this comparison, since data statistics are closer to the ones in Huggett et al. (2006).

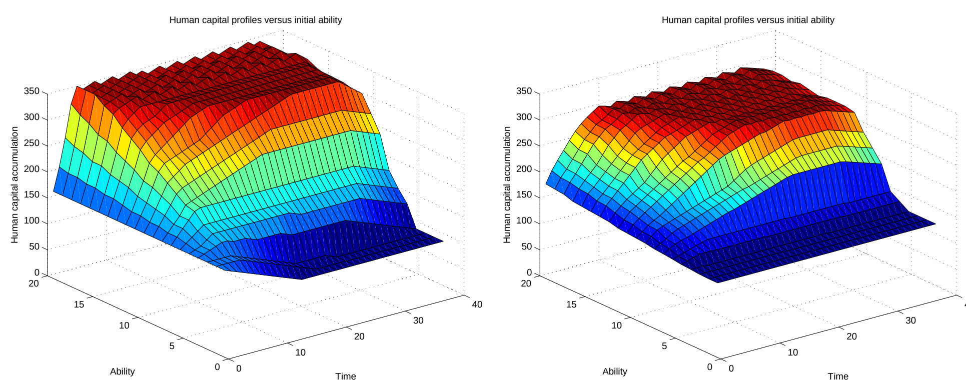
This is because optimal accumulation within the standard model dictates that, early in the life-cycle, agents with high learning ability devote most or all of their time to accumulating human capital (see Huggett et al. (2006) for details). My version allows for more productive human capital accumulation in college. The ability to learn significantly contributes to human capital production in this period. Consequently, lower initial human capital levels and higher ability levels match life-cycle earnings and the correlation between human capital and learning ability is lower when college enrollment is endogenized.

Table A-1: Joint Distribution of Ability and Human Capital for High School Graduates

Characteristic	Statistic	My Model	BP Model
Ability	Mean(a)	0.301	0.259
	Coef of Variation (a)	0.517	0.456
	Skewness(a)	1.687	1.464
Human capital	Mean(h)	74.691	91.7
	Coef of Variation (h)	0.527	0.475
	Skewness(h)	1.729	1.53
	Correlation (a,h)	0.769	0.852

Figures A-3 and A-4 present life-cycle profiles for human capital accumulation by education groups. The first set of graphs represents the prediction of the model when heterogeneity in human capital is absent and the second set when heterogeneity in ability levels is absent. Both cases are discussed in the paper in Section 4.1.2.

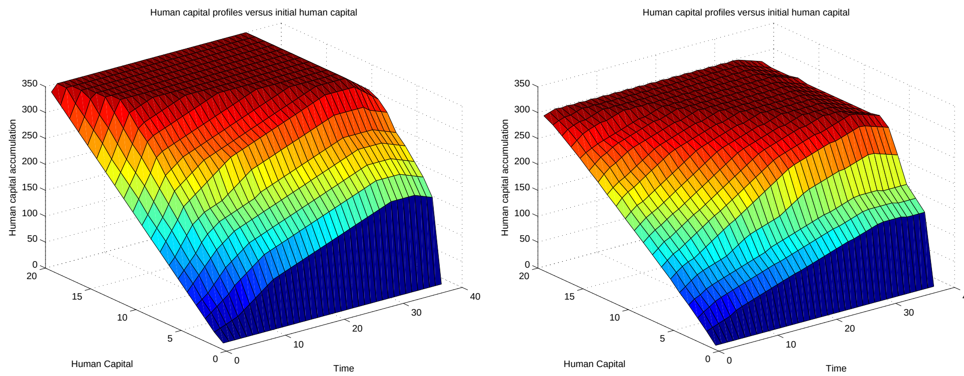
Figure A-3: Human Capital Accumulation By Education Group



(a) College (b) No college

Note: Figures are drawn across ability levels (with low to high levels from 0 to 20), for a fixed level of initial human capital and assets.

Figure A-4: Human Capital Accumulation By Education Group



(a) College　　(b) No college

Note: Figures are drawn across initial human capital levels (with low to high levels from 0 to 20), for a fixed level of ability and initial assets.

References

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